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On the Possible Enhancement of the Magnetic Field by Neutrino Reemission Processes in the Mantle of a Supernova

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Abstract

URCA neutrino reemission processes under the conditions in the mantle of a supernova with a strong toroidal magnetic field are investigated. It is shown that parity violation in these processes can be manifested macroscopically as a torque that rapidly spins up the region of the mantle occupied by such a field. Neutrino spin-up of the mantle can strongly affect the mechanism of further generation of the toroidal field, specifically, it can enhance the field in a small neighborhood of the rigid-body-rotating core of the supernova remnant.

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The problem of the shedding of the mantle in an explosion of a type-II supernova is still far from a complete solution [1]. It is known that in several seconds after the collapse of a presupernova an anomalously high neutrino flux with typical luminosities $L \sim 10^{52}$ ergs/s is emitted from the neutrinosphere, which

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is approximately of the same size as the remnant core [2]. In principle, such a neutrino flux could initiate a process leading to the shedding of the mantle as a result of the absorption and scattering of neutrinos by nucleons and the e^+e^- plasma of the medium [3]. However, detailed calculations in spherically symmetric collapse models have shown that such processes are too weak for mantle shedding [4]. In a magnetorotational model [5], mantle shedding is initiated by the outward pressure of a strong toroidal magnetic field generated by the differential rotation of the mantle with the core's primary poloidal magnetic field "frozen" into it. Indeed, as calculations show [6], when a mantle rotates with a millisecond period in a poloidal field $B_0 \sim 10^{12} - 10^{13}$ G a toroidal field $B \sim 10^{15} - 10^{16}$ G is generated in a time of order of a second. We note that the model in Ref. [5] contains a fundamental limitation on the energy of the toroidal field (it cannot exceed the kinetic energy of the "core + shell" system) and therefore on the maximum field itself.

In the present letter we investigate the possibility that this magnetic field "frozen in" the mantle is enhanced as the result of elementary neutrino reemission processes occurring in the mantle. We assume the mantle in the vicinity of the neutrinosphere to be a hot ($T \sim$ several MeV) and quite dense (though transparent to neutrinos, $\rho \sim 10^{11} - 10^{12}$ g/cm^3) medium consisting of free nucleons and e^+e^- plasma. Under these conditions the dominant neutrino reemission processes are the URCA processes:

$$p + e^- \to n + \nu_e \ , \tag{1}$$

$$n + e^+ \to p + \tilde{\nu}_e \ , \tag{2}$$

$$n + \nu_e \to p + e^- \ , \tag{3}$$

$$p + \tilde{\nu}_e \to n + e^+ \ . \tag{4}$$

We note that β decay is statistically suppressed in such a medium. The basic idea of this letter is as follows. In an external magnetic field, neutrinos are emitted and absorbed asymmetrically with respect to the direction of the magnetic field as a result of the parity violation in the processes (1)–(4) [7]. Therefore a macroscopic torque spinning up the mantle can arise in a toroidal field. It is known [8] that for an equilibrium neutrino distribution function such a neutrino-recoil momentum must be zero. However, the supernova region under consideration is nonequilibrium for neutrinos, so that the torque that arises in it is different from zero. Moreover, as we shall show below, the torque can be large enough to change substantially the distribution of the angular

rotational velocities of the mantle in the region filled with a strong magnetic field over the characteristic neutrino emission times. According to the equation governing the generation of a toroidal field [6], a large change in the gradient of the angular velocities in the region can lead to redistribution of the magnetic field (specifically, enhancement of the field in a small neighborhood of the rigid-body-rotating core).

A quantitative estimate of the effect follows from the expression for the energy-momentum transferred by neutrinos to a unit volume of the mantle per unit time:

$$\frac{dP_\alpha}{dt} = \frac{1}{V}\int \prod_i dn_i f_i \prod_f dn_f (1-f_f)\frac{|S_{if}|^2}{\mathcal{T}} k_\alpha, \tag{5}$$

where dn_i and dn_f are the number of initial and final states in an element of the phase space, f_i and f_f are the distribution functions of the initial and final particles, k_α is the neutrino momentum, $|S_{if}|^2/\mathcal{T}$ is the squared S-matrix element per unit time. It is of interest to calculate the latter under our conditions, since, as far as we know, the URCA processes (1)–(4) have been previously studied for relatively weak [9] ($B \lesssim m_e^2/e$) and very strong [10] ($B \sim m_p^2/e$) fields. Assuming that electrons and positrons in the plasma mainly occupy only the lowest Landau level ($\mu_e \lesssim \sqrt{2eB}$, where μ_e is the chemical potential of electrons), we obtained the following expression for the squared S-matrix element summed over all proton Landau levels and polarizations of the final particles and averaged over the polarizations of the initial particles:

$$\begin{aligned} |S_{if}|^2 &= \frac{G_F^2 \cos^2\theta_c (2\pi)^3\, \mathcal{T}}{2L_y L_z V^2}\,\frac{\exp(-Q_\perp^2/2eB)}{4\omega\varepsilon} \\ &\times \frac{1}{2}\left[\sum_{n=0}^{\infty}\frac{|M_+|^2}{n!}\left(\frac{Q_\perp^2}{2eB}\right)^n \delta^{(3)} + \sum_{n=1}^{\infty}\frac{|M_-|^2}{(n-1)!}\left(\frac{Q_\perp^2}{2eB}\right)^{n-1}\delta^{(3)}\right], \end{aligned} \tag{6}$$

$$\begin{aligned} |M_\sigma|^2 &= 4(1+g_a\sigma)^2\Big[2(up)(uk) - (pk) - (up)(k\tilde{\varphi}u) - (uk)(p\tilde{\varphi}u)\Big] \\ &+ 8g_a^2(1+\sigma)\Big[(pk) - (p\tilde{\varphi}k)\Big], \end{aligned} \tag{7}$$

where u_α is a four-velosity of a medium, $\mathbf{B} = (0,0,B)$, and $\delta^{(3)}$ is the production of the energy delta-function and two delta-functions of the momentum in the direction of the magnetic field and one transverse momentum component which

are conserved in the reactions; $(pk) = \varepsilon\omega - p_3 k_3$, $p^\alpha = (\varepsilon, \mathbf{p})$ and $k^\alpha = (\omega, \mathbf{k})$ are the four-momenta of the electron and neutrino, $Q_\perp^2$ is the square of the momentum transfer transverse to the to the magnetic field in the reactions (1) and (3) [with the corresponding substitutions $p \to -p$ and $k \to -k$ in the crossing reactions (2) and (4)], $\tilde{\varphi}_{\alpha\beta} = \tilde{F}_{\alpha\beta}/B$ is the dimensionless dual magnetic-field tensor, $\sigma = \pm 1$ is the projection of the proton spin on the direction of the magnetic field, n is the summation index over the proton Landay levels, $\mathcal{T}V = \mathcal{T}L_x L_y L_z$ is the normalization four-volume, g_a is the axial constant of the nucleonic current, G_F is the Fermi constant, and θ_c is the Cabibbo angle. We note that in the limit of a strong magnetic field, when the protons occupy only the ground Landay level, expressions (6) and (7) agree with the result obtained previosly in Ref. [10].

Analysis shows that URCA processes are the fastest reactions in medium considered, and they transfer the medium into a state of β equilibrium in a time of order 10^{-2} s. Therefore we employed the condition of β equilibrium and singled out in the expression for the energy-momentum transfer to the shell (5) the separate contributions from processes involving neutrinos (1), (3) and antineutrinos (2), (4):

$$\frac{dP_\alpha^{(\nu,\tilde{\nu})}}{dt} = \int \frac{d^3k}{(2\pi)^3}\; k_\alpha \left[1 + \exp\left(\frac{-\omega + \mu_{(\nu,\tilde{\nu})}}{T}\right)\right] \mathcal{K}^{(\nu,\tilde{\nu})}\, \delta f^{(\nu,\tilde{\nu})} . \tag{8}$$

Here, $\delta f^{(\nu,\tilde{\nu})}$ is the deviation of the distribution function from the equilibrium function, $\mathcal{K}^{(\nu,\tilde{\nu})}$ is the (anti)neutrino absorption coefficient, defined as

$$\mathcal{K}^{(\nu,\tilde{\nu})} = \int \prod_i dn_i f_i \prod_f dn_f (1 - f_f) \frac{|S_{if}|^2}{\mathcal{T}} , \tag{9}$$

where the integration extends over all states except the neutrino states in the reaction (3) and antineutrino states in the reaction (4), respectively. As follows from Eq. (8), actually, the momentum transferred to the medium is different from zero only if the neutrino distribution function deviates from the equilibrium distribution.

To calculate the absorption coefficient $\mathcal{K}^{(\nu,\tilde{\nu})}$ we assumed that the ultrarelativistic e^+e^- plasma occupies only the ground Landau level, while the protons occupy quite many levels (the dimensionless parameter $\delta = eB/m_pT \ll 1$). We also used the fact that at the densities under consideration the nucleonic gas is Boltzmannian and nonrelativistic. Then, dropping terms $\sim \delta$, we can write

expression (9) in the form

$$\mathcal{K}^{(\nu,\tilde{\nu})} = \frac{G_F^2 \cos^2\theta_c \, eB \, N_{(n,p)}}{2\pi}\Big[(1+3g_a^2) - (g_a^2-1)k_3/\omega\Big] \times \left[1+\exp\left(\frac{\pm(\mu_e - \triangle) - \omega}{T}\right)\right]^{-1}, \tag{10}$$

where N_n, N_p, and m_n, m_p are the number densities and masses of the neutrons and protons, respectively, $\triangle = m_n - m_p$, and ω and k_3 are the neutrino energy and the neutrino momentum in the direction of the magnetic field, respectively.

For further calculations we employed the neutrino distribution function in the model of a spherically symmetric collapse of a supernova in the absence of a magnetic field [11]. This is a quite good approximation when the region occupied by the strong magnetic field is smaller than or of the order of the neutrino mean-free path. By the strong field we mean the field in which e^+e^- plasma occupies only the ground Landau level: $eB \gtrsim \mu_e^2$. In the model of Ref. [6] the region occupied by such a field is no greater than several kilometers in size, and we estimate the neutrino mean-free path in this region as

$$l_\nu \simeq 4\,\mathrm{km}\left(\frac{4.4\times 10^{16}\,\mathrm{G}}{B}\right)\left(\frac{5\times 10^{11}\mathrm{g/cm}^3}{\rho}\right). \tag{11}$$

Therefore the magnetic field cannot strongly alter the neutrino distribution function, and our approximation is quite correct.

As calculations of the components of the energy-momentum (8) transferred to the medium during neutrino reemission showed, the radial force arising is much weaker than the gravitational force and cannot greatly influence the mantle dynamics. However, the force acting in the direction of the magnetic field can change quite rapidly the distribution of the angular velocities in the region occupied by the strong magnetic field. The density of this force can be represented as

$$\Im_\parallel^{(tot)} = \Im_\parallel^{(\nu)} + \Im_\parallel^{(\tilde{\nu})} = \mathcal{N}\Big[\Big(3\left\langle\mu^2\right\rangle_\nu - 1\Big)I(a)e^{-a} + \Big(3\left\langle\mu^2\right\rangle_{\tilde{\nu}} - 1\Big)I(-a)\Big], \tag{12}$$

$$\left\langle\mu^2\right\rangle = \left(\int \mu^2\,\omega\,f\,d^3k\right)\cdot\left(\int \omega\,f\,d^3k\right)^{-1},$$

where μ is the cosine of the angle between the neutrino momentum and the radial direction, $a = \mu_e/T$, and

$$I(a) = \int\limits_0^\infty \frac{y^3\, dy}{\mathrm{e}^{y-a} + 1} \, .$$

In deriving Eq. (12) we use the one-dimensional factorized neutrino distribution function $f^{(\nu,\tilde{\nu})} = \phi^{(\nu,\tilde{\nu})}(\omega/T_\nu)\, \Phi^{(\nu,\tilde{\nu})}(r,\mu)$ [1], where T_ν is the neutrino spectral temperature and r is the distance from the core center. To estimate the force in the diffusion region we assumed that $T_\nu \simeq T$ and chose $\phi^{(\nu,\tilde{\nu})}(\omega/T_\nu) = \exp\left(-\omega/T_\nu\right)$. We determined the dimensional parameter $\mathcal{N}$ in expression (12) as

$$\begin{aligned} \mathcal{N} &= \frac{G_F^2 \cos^2\theta_c}{(2\pi)^3} \frac{g_a^2 - 1}{3}\, eB\, T^4\, N_N \qquad (13) \\ &\simeq 4.5 \times 10^{20}\, \frac{\text{dynes}}{\text{cm}^3} \left(\frac{T}{5 \text{ MeV}}\right)^4 \left(\frac{B}{4.4 \times 10^{16} \text{ G}}\right) \left(\frac{\rho}{5 \times 10^{11} \text{ g/cm}^3}\right) \, , \end{aligned}$$

where $N_N = N_n + N_p$ is the total nucleon number density.

The force (12) was estimated numerically in the diffusion region of the supernova atmosphere for typical (excluding the field) values of the macroscopic parameters for this region: $T = 5$ MeV, $B = 4.4 \times 10^{16}$ G, $\rho = 5 \times 10^{11}$ g/cm^3. For these values $a \simeq 3$, $\left\langle \mu^2 \right\rangle_\nu \simeq \left\langle \mu^2 \right\rangle_{\tilde{\nu}} \simeq 0.4$ (Ref. [11]), and the force density in the direction of the field can be estimated from Eq. (12) as

$$\Im_\parallel^{tot} \simeq \Im_\parallel^\nu \simeq \mathcal{N} \, . \qquad (14)$$

We note that the angular acceleration produced by the torque exerted by such a force is large enough to spin up the region of the mantle containing a strong magnetic field to typical angular velocities of a fast pulsar (with the rotational period $P_0 \sim 10^{-2}$ s) in a characteristic time of order of a second. In our opinion, this result is of interest in itself and can serve as a basic for a number of applications. However, we shall give a qualitative discussion of only one possible manifestation of this result – the effect of such a fast spin-up of the mantle on the further generation of the toroidal magnetic field. Indeed, if the modification of the gradient of the angular velocities of the mantle is large, the toroidal magnetic field in the mantle at subsequent times will vary according to a law that is different from the linear law [6]. Analysis of the equation governing

the generation of a toroidal field with allowance for the force (14), which is linear in this field, leads to the conclusion that its growth in time is much faster (exponential) in the quite small region in which the force acts ($eB \gtrsim \mu_e^2$). However, the main source of the magnetic field energy, just as in the case when there is no force, is the kinetic energy of the rigid-body-rotating core. Thus the force (14) can lead to a peculiar rearrangement of the region occupied by the strong field. Specifically, with virtually no change in energy, the magnetic field can become concentrated in a narrower spatial region and can therefore have in this region higher intensities, on average, than in the absence of the spin-up force. We note that the effect under discussion can strongly influence the mantle-shedding process and also the mechanism leading to the formation of an anisotropic γ-ray burst in the explosion of a supernova with a rapidly rotating core [12]. However, in order to perform detailed calculations of the generation of a toroidal field, the enhancement of the field due to the "neutrino spin-up", and the effect of this enhancement on the indicated processes, it is necessary to analyze the complete system of MHD equations. That analysis lies far outside the scope of the present work and is a subject of a separate investigation. Qualitative estimates show that the magnetic field with the strength $B \sim 10^{17}$ G can be generated by the above-described mechanism in a small neighborhood (of order a kilometer) of a rapidly rotating core with the period $P_0 \sim 5 \times 10^{-3}$ s, and this region decreases with increasing the period.

In summary, we have shown that URCA neutrino reemission processes can produce, in the region of the mantle that is filled with a strong toroidal magnetic field, angular accelerations which are sufficiently large as to greatly influence the mechanism of further generation of the field. Specifically, such rapid redistribution of the angular velocities can enhance the field in a small neighborhood of the rigid-body-rotating core of a remnant.

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